NAME : KEY

EXAM

EXAM 1 EQUATION SHEET

CHAPTER 15

$|\vec{F}_e| = k_e \frac{|q_1 q_2|}{r^2}$ $\quad \vec{F}_e = q\vec{E}$ $\quad \vec{F}_{NET} = \vec{F}_1 + \vec{F}_2 + \ldots$

$|\vec{E}| = k_e \frac{|q|}{r^2}$ $\quad \vec{E}_{NET} = \vec{E}_1 + \vec{E}_2 + \ldots$ $\quad k_e = 8.99 \times 10^9 \frac{Nm^2}{C^2}$

CHAPTER 16

$W = \Delta KE$ $\quad W = -\Delta PE$ $\quad W = qE_x \Delta x$ $\quad \Delta PE = q\Delta V$ $\quad \Delta V = -E_x \Delta x$

$V = k_e \frac{q}{r}$ $\quad PE = k_e \frac{q_1 q_2}{r}$ $\quad C = \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{d}$ $\quad \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

ENERGY STORED $= \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}$

$C_{NET} = C_1 + C_2 + \ldots$ (PARALLEL) $\quad \frac{1}{C_{NET}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$ (SERIES)

CHAPTER 17

$I = \frac{\Delta Q}{\Delta t}$ $\quad \Delta V = IR$ $\quad R = \rho \frac{l}{A}$ $\quad R = R_0[1 + \alpha(T - T_0)]$

$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$ $\quad \rho = \rho_0[1 + \alpha(T - T_0)]$

CHAPTER 18

$\Delta V = \mathcal{E} - Ir$

$R_{NET} = R_1 + R_2 + \ldots$ (SERIES)

$\frac{1}{R_{NET}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$ (PARALLEL)

$\tau = R_{NET} C$

$q = Q_{MAX}(1 - e^{-t/RC})$ [CHARGING]

$q = Q_{MAX} e^{-t/RC}$ [DISCHARGING]

OTHERS

$\sum F = ma$ $\quad 1\,W = 1\,J/s$

$F_g = mg$ $\quad 1\,\Omega = 1\,V/A$

$1e = 1.6 \times 10^{-19}\,C$ $\quad 1\,V = 1\,J/C$

$1\,F = 1\,C/V$ $\quad 1\,A = 1\,C/s$

$1\,J = 1\,N \cdot m$ $\quad A = \pi \frac{d^2}{4} = \pi r^2$

$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

[1] A special toaster rated at 900 W operates on a 120-V household circuit and a 3.5 m length of platinum wire as its heating element. The operating temperature of this element is 240°C. What is the cross-sectional area of the wire? Use platinum's values of $\rho_0=11\times10^{-8}\ \Omega m$ and $\alpha=3.92\times10^{-3}\ 1/°C$.

$$P=\frac{(\Delta V)^2}{R} \Longrightarrow R=\frac{(\Delta V)^2}{P}=\frac{(120\ V)^2}{900\ W}=16\Omega$$

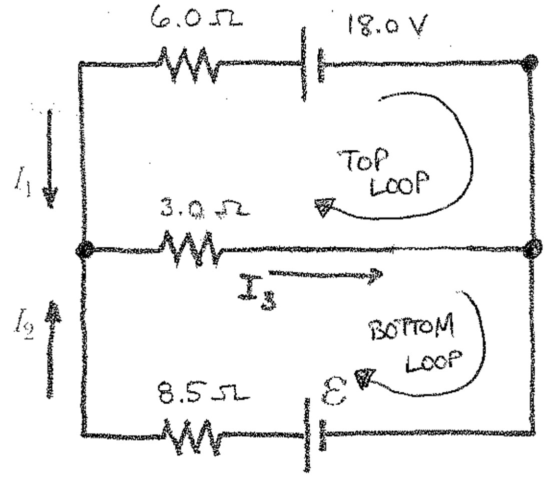
$$R=\rho\frac{l}{A} \Longrightarrow A=\frac{\rho l}{R}=\frac{\rho_0[1+\alpha(T-T_0)]l}{R}$$

$$A=\frac{(11\times10^{-8}\Omega m)[1+(3.92\times10^{-3}\frac{1}{°C})(240°C-20°C)](3.5m)}{16\Omega}$$

$$A=\frac{(11\times10^{-8}\Omega m)[1+0.862](3.5m)}{16\Omega}$$

$$\boxed{A=4.5\times10^{-8}\ m^2}$$

[2] If I_3= 3.5 A, then find I_1, I_2, and $\mathcal{E}$.



• TOP LOOP, STARTING CLOCKWISE FROM THE TOP RIGHT CORNER:

$$+\Delta V_{3\Omega} + \Delta V_{6\Omega} - 18\text{ V} = 0$$

$$I_3 R_{3\Omega} + I_1 R_{6\Omega} - 18\text{ V} = 0$$

$$(3.5\text{ A})(3\Omega) + I_1(6\Omega) - 18\text{ V} = 0$$

$$I_1 = \frac{18\text{ V} - 10.5\text{ V}}{6\Omega} = \boxed{1.25\text{ A} = I_1}$$

• JUNCTION RULE AT LEFT JUNCTION:

$$I_1 + I_2 = I_3 \Longrightarrow I_2 = I_3 - I_1 = 3.5\text{ A} - 1.25\text{ A} = \boxed{2.25\text{ A} = I_2}$$

• BOTTOM LOOP, STARTING CLOCKWISE FROM THE MIDDLE RIGHT JUNCTION:

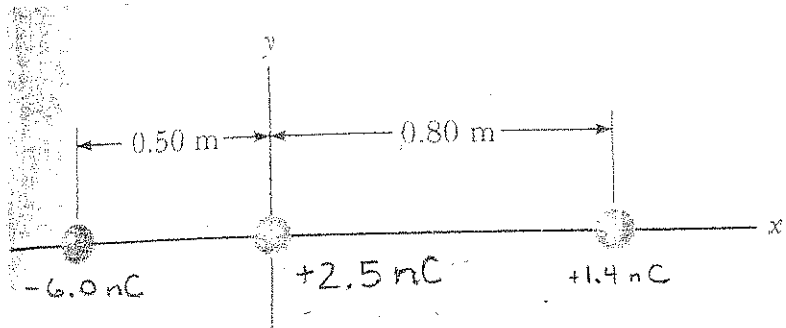
$$+\mathcal{E} - \Delta V_{8.5\Omega} - \Delta V_{3\Omega} = 0$$

$$+\mathcal{E} - I_2 R_{8.5\Omega} - I_3 R_{3\Omega} = 0$$

$$+\mathcal{E} - (2.25\text{ A})(8.5\Omega) - (3.5\text{ A})(3\Omega) = 0$$

$$\mathcal{E} = 19.1\text{ V} + 10.5\text{ V} = \boxed{29.6\text{ V} = \mathcal{E}}$$

[3] Three point charges are aligned along the x-axis as shown. Find the electric field when x=+3.5m and y=0.



$$E_1 = k_e \frac{|q_1|}{r_1^2} = \left(8.99\times10^{9}\ \frac{Nm^2}{C^2}\right)\frac{|-6.0\times10^{-9}\,C|}{(4.0m)^2} = -3.37\ \frac{N}{C}$$

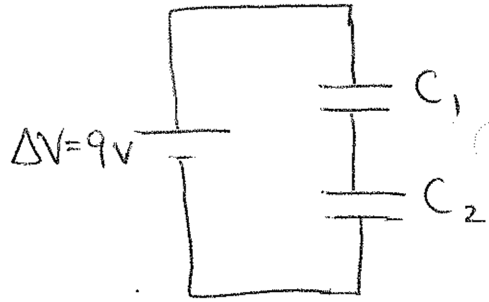
$$E_2 = k_e \frac{|q_2|}{r_2^2} = \left(8.99\times10^{9}\ \frac{Nm^2}{C^2}\right)\frac{|+2.5\times10^{-9}\,C|}{(3.5m)^2} = +1.83\ \frac{N}{C}$$

$$E_3 = k_e \frac{|q_3|}{r_3^2} = \left(8.99\times10^{9}\ \frac{Nm^2}{C^2}\right)\frac{|+1.4\times10^{-9}\,C|}{(2.7\ m)^2} = +1.73\ \frac{N}{C}$$

$$E_{NET} = E_1 + E_2 + E_3 = -3.37\frac{N}{C} + 1.83\frac{N}{C} + 1.73\frac{N}{C}$$

$$\boxed{E_{NET} = +0.19\ \frac{N}{C}}$$

[4] Two capacitors, C_1=16 μF and C_2=24 μF, are connected in series, and a 9-V battery is connected across them.



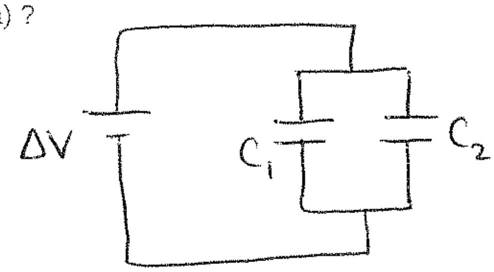
(a) Find the net capacitance and the total energy stored.

$$\frac{1}{C_{NET}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{16\mu F} + \frac{1}{24\mu F} = \frac{5}{48\mu F}$$

$$\boxed{C_{NET} = 9.6\,\mu F}$$

$$\text{ENERGY STORED} = \frac{1}{2} C_{NET} (\Delta V)^2 = \left(\frac{1}{2}\right)(9.6\times10^{-6} F)(9V)^2 = \boxed{3.89\times10^{-4} J = E}$$

(b) If the same capacitors are connected in parallel, what potential difference would be required across the so that the combination stores the same amount of energy as in part (a) ?



$$C_{NET} = C_1 + C_2 = 16\mu F + 24\mu F = 40\mu F$$

$$E = \frac{1}{2} C_{NET} (\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2E}{C_{NET}}} = \sqrt{\frac{(2)(3.89\times10^{-4} J)}{40\times10^{-6} F}}$$

$$\boxed{\Delta V = 4.41\ V}$$

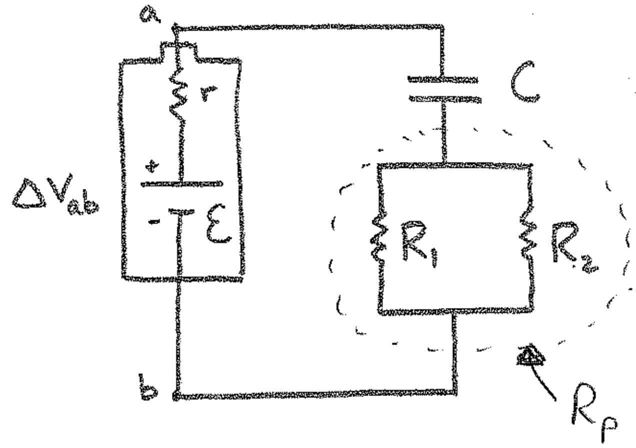
(c) In this new parallel circuit, what is the total energy stored in each capacitor?

$$E = \frac{1}{2} C (\Delta V)^2$$

$$E_{16} = \left(\frac{1}{2}\right)(16\times10^{-6} F)(4.41\ V)^2 = \boxed{1.56\times10^{-4} J = E_{16}}$$

$$E_{24} = \left(\frac{1}{2}\right)(24\times10^{-6} F)(4.41\ V)^2 = \boxed{2.33\times10^{-4} J = E_{24}}$$

[5] An uncharged capacitor, C = 30 μF, is connected in series to two resistors, R_1 = 50 Ω and R_2 = 20 Ω, that are connected in parallel as shown.



(a) If this combination is connected to a battery with a terminal voltage of 11.5 V and an internal resistance of r = 4 Ω, then what is the net resistance of the circuit?

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{50\Omega} + \frac{1}{20\Omega} = \frac{7}{100\Omega}$$

$$R_P = 14.3\,\Omega$$

$$R_{NET} = R_P + r = 14.3\,\Omega + 4\,\Omega$$

$$\boxed{R_{NET} = 18.3\,\Omega}$$

(b) What is the time constant of the circuit?

$$\tau = R_{NET}C = (18.3\,\Omega)(30\times10^{-6}\,F) = \boxed{5.5\times10^{-4}\,s = \tau}$$

(c) What is the EMF of the battery?

$$\Delta V = \mathcal{E} - Ir \Rightarrow \mathcal{E} = \Delta V + Ir$$

$$\Delta V = IR_P \Rightarrow I = \frac{\Delta V}{R_P}$$

$$\therefore\ \mathcal{E} = \Delta V + \frac{\Delta V}{R_P}\cdot r = 11.5\,V + \frac{11.5\,V}{14.3\,\Omega}\cdot 4\,\Omega = 11.5\,V + 3.2\,V$$

$$\boxed{\mathcal{E} = 14.7\,V}$$

1	2	3	4	5	TOTAL